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Kaluza–Klein Space and Strong Gravity

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A study of strong gravity theory is carried out in five-dimensional Kaluza–Klein space-time. A relation between the cosmological constant and the radius parameter of the fifth dimension is obtained. The effect of the extra dimension is seen through the generation of masses simulating a Regge-like mass spectrum. It is found that the confinement mechanism is built into the strong gravity formalism. We also discuss the trapping of quarks in the 5D background.

1. INTRODUCTION

There have been some attempts to understand the hadronic interaction based on a space-time description. In the geometrodynamics of Preparata (Preparata and Craigie, 1976) hadron dynamics follows from space-time geometry. There is also a model (Schrempp and Schrempp, 1977) of a hadron–hadron scattering where hadronic diffraction is considered as a tunneling phenomenon. The geometry of the hadron in the model turns out to be identical to the one found in the color confinement scheme. In the most remarkable approach in this direction (Wess and Zumino, 1970; Salam *et al.*, 1971; Sivaram and Sinha, 1974, 1975, 1979; Biswas *et al.*, 1983), called the strong gravity formalism, a strongly interacting spin-2^+ meson lying on a pomeron trajectory is described by an equation similar to the Einstein field equation with $f^{\mu\nu}$ (2^+ meson mediating field) replacing the graviton field $g^{\mu\nu}$. There is f–g mixing to give mass to the f-quanta.

In this paper we extend this two-tensor theory of gravitation to higher dimensions, particularly restricting ourselves to five-dimensional Kaluza–Klein space. All the models of strong gravity theory so far constructed are built on four-dimensional Riemannian space-time with an Einstein–Hilbert type of action; the cosmological constant is added to generate the Λr^2-type potential of quantum chromodynamics. In supergravity theories extra

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dimensions have been introduced to incorporate internal symmetry and have been treated at par with ordinary four dimensions, but the strong gravity theory in Kaluza-Klein space has not been exploited so far. When normal gravitation is seen through Kaluza-Klein space, we get a unification of electromagnetism and gravitation. The radius of the extra dimensional compact space in this type of theory turns out to be of the order of the Planck length 10^{-33} cm, resulting in a mass scale of 10^{19} GeV. Before going on to describe the salient features of f-g theory in 5D space, we note that the f-g theory in four space-time dimensions cannot explain the origin of the cosmological constant and also is unable to give a kinematic theory of mass. The cosmological constant is necessary to explain the confining aspects of the solitonic solutions of strong gravity theory, whereas a kinematic theory of mass is needed to explain the origin of the Regge states of hadronic physics. Moreover, the mechanism of mass generation remains unclear in all these four-dimensional f-g theories. In our attempt, we find that (i) the f-g theory in five-dimensional Kaluza-Klein space constructed from an Einstein-type action without cosmological constant is equivalent to the f-g theory in four dimensions with a cosmological constant, (ii) the massive modes of the theory generate the cosmological constant, (iii) the extra dimensional compact space generates masses characteristic of strong interaction provided we fix the radius of the compact circle to be of the order of 1 F, (iv) Regge states are simulated, signaling the confining aspects of the theory, and (v) a fluctuation of the radius of the extra dimensional compact circle generates mass at the scale of hadronic physics, namely ~1 GeV.

In Section 2 we describe the f-g theory in five-dimensional Kaluza-Klein space; Section 3 deals with the derivation of the cosmological constant. Section 4 deals with the mass scales of the theory, elucidating its connection with the Regge states of hadronic physics. Section 5 deals with the trapping of quarks and gluons in a given strong gravitational background, and in Section 6 we report some of our results on the mass generation in a five-dimensional space-time. We end with a concluding section.

2. f–g THEORY IN FIVE DIMENSIONS

We take, as in the original Kaluza-Klein space, the metric ground state to be a product space $M^4 \times S^1$, where S^1 is a circle characterized by a linear variable ranging from 0 to $2\pi R_s$, with R_s the radius of the S^1 circle. The ground state is now described by

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - d\phi^2 \tag{1}$$

Now we assume that in the vicinity of the hadron, due to strong interaction, the metric is f_{AB}, where

$$ds^2 = f_{AB}\, dx^A\, dx^B \tag{2}$$

and A, $B = 0, 1, 2, 3, 5$. The Einstein-Hilbert action of the five-dimensional f–g theory is now written as

$$S = -\frac{1}{16\pi G_N}\int d^5x\,(-g)^{1/2}R(g) - \frac{1}{16\pi G_s}\int d^5x\,(-fe)^{1/2}R(f) \tag{3}$$

where $R(g)$ and $R(f)$ are Ricci scalars constructed from g_{AB} and f_{AB}, respectively. If we consider $G_s \gg G_N$, we have the dominant term

$$S = \frac{1}{16\pi G_s}\int d^5x\,(-f)^{1/2}R(f)$$

As the extradimensional space is compact, it is possible to expand $f_{AB}(x^\mu, \phi)$ into Fourier series as

$$f_{AB}(x^\mu, \phi) = \sum_{n=-\infty}^{+\infty} f_{AB}^{(n)}(x^\mu)\, e^{inx^5/R_s} \tag{4}$$

Under suitable gauge, each component $f_{AB}^{(n)}$ will satisfy

$$\Box^5 f_{AB}^{(n)} = 0$$

or

$$\left(\frac{\partial^2}{\partial t^2} - \sum_i \frac{\partial^2}{\partial x_i^2} + m_n^2\right) f_{AB}^{(n)} = 0$$

where m_n is given by

$$m_n^2 = \frac{n^2}{R_s^2} \tag{5}$$

In usual Kaluza-Klein theory $R_s \simeq 10^{-33}$ cm, resulting in a mass scale of 10^{19} GeV. As experiments are conducted far below this energy range, one takes an average over the fifth dimension and works with only the zeroth mode ($n = 0$) of the theory. But in our approach the mass scale is ~ 1 GeV, corresponding to $R_s \sim 10^{-14}$ cm, and we have to consider all the modes of the theory. Let us first see the effect of the zeroth mode, with

$$f_{AB} = \left(\begin{array}{c|c} \bar{f}_{\mu\nu} + A_\mu A_\nu & A_\mu \\ \hline A_\nu & 1 \end{array}\right) \tag{6}$$

$$f^{AB} = \left(\begin{array}{c|c} \bar{f}^{\mu\nu} & -A^\mu \\ \hline -A^\nu & 1 + A_\lambda A^\lambda \end{array}\right) \tag{7}$$

We have omitted the superscript $n=0$ on the metric f_{AB}. With this form of f_{AB}, we get

$$ds^2 = f_{\mu\nu}\, dx^\mu\, dx^\nu + (dx^5 + A_\mu\, dx^\mu)^2 \tag{8}$$

$$\det(f_{AB}) = \det(\bar{f}_{\mu\nu}) \equiv f(4) \tag{9}$$

To obtain the Einstein-like field equation of strong gravity, it is required to calculate the Ricci scalar $R^{(5)}(f)$ in five dimensions. It can be shown that

$$R^{(5)} = R^{(4)} + \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{10}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{11}$$

and $R^{(4)}$ is a Ricci scalar evaluated with the metric $\bar{f}^{\mu\nu}$. As $F_{\mu\nu}$ and A_μ do not depend on an internal coordinate, we get from (3) after integrating over $x^5 = \phi$

$$S = -\frac{1}{16\pi G_f} \int d^4x\, [-f^{(4)}]^{1/2} (R^{(4)} + \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}) \tag{12}$$

where $G_f = G_s/2\pi R_s$ is now identified with the strong gravitational constant, i.e., $G_f = 10^{38} g_N$. To fix R_s in our approach, we consider a scalar field with action

$$S_\phi = \int d^5x\, [-f^{(5)}]^{1/2} (\partial_A \Phi)(\partial_B \Phi^\dagger) f^{AB} \tag{13}$$

Again assuming

$$\Phi = \frac{1}{(2\pi R_s)^{1/2}} \sum_n \Phi^{(n)}(x^\mu)\, e^{inx^5/R_s}$$

we get

$$\partial_A \Phi\, \partial_B \Phi^\dagger f^{AB} = \left| \left(\partial_\mu + i \frac{n}{R_s} A_\mu \right) \Phi \right|^2 + \frac{n^2}{R_s^2} |\Phi|^2$$

so that the Fourier component of Φ behaves as a particle of charge q and mass M_n, where

$$q = (16\pi G_s)^{1/2} \frac{n}{R_s}, \qquad M_n = \frac{n}{R_s} \tag{14}$$

Thus

$$\begin{aligned} \alpha &= \frac{e^2}{4\pi\hbar c} = \frac{4\hbar G_s}{c^3 R_s^2} \\ R_s &= \frac{2}{\alpha^{1/2}} \left(\frac{\hbar G_s}{c^3} \right)^{1/2} = \frac{2}{\alpha^{1/2}} \left(\frac{\hbar G_N}{c^3} \right)^{1/2} \times 10^{19} \\ &= 3.7 \times 10^{-13}\ \text{cm} \end{aligned} \tag{15}$$

Thus, we see that even in the hadronic world, a Kaluza–Klein type description will work, with the result that the radius of the extra dimensional compact circle coincides with the dimension of the hadron. This peculiar property of f-gravity theory in five dimensions is the root that will generate the confining properties in the theory. A straightforward approach is to construct solitonic solutions in Kaluza–Klein theories. By solitons we mean nonsingular solutions of the classical field equations which represent spatially localized lumps that are topologically stable. As we have discussed only the zeroth-order mode and hadrons are viewed from a four-dimensional point of view having masses $\sim n/R_s$, we will be interested only in those solutions of classical field equations where proper account of the higher order modes has been taken seriously. So the solitonic solutions of $n=0$ mode theory of f-gravity will not be much help in our approach.

3. COSMOLOGICAL CONSTANT AND $N\neq 0$ MODES

It would be worthwhile to discuss some salient features arising out of equation (15). The structure of the internal space is not clear in the Kaluza–Klein approach of normal gravity except that it is determined by the fine structure constant α and G_N, the normal gravitational constant. The internal space manifests itself only through the massive modes $\sim 10^{19}$ GeV, which is extremely large, so that we take only an average over the fifth dimension. As the strong interaction is the strongest of all the known four interactions and the radius for the compact circle $\sim 10^{-13}$ cm is the largest, the internal space still remains hidden within observed particles and it is a surprise that it is of the order of hadronic dimensions. So it is quite possible to have a theory where the radius of the S^1 circle is a free parameter such that

$$\frac{G_N}{R_P^2}=\frac{G_i}{R_i^2} \tag{16}$$

where G_i and R_i are, respectively, the coupling constant and radius of the compact space corresponding to the new theory; G_N is the normal gravitational constant and R_P is the Planck length ($\simeq 10^{-33}$ cm) of normal gravity. Equation (16) will make the fine structure constant to be equal to the present value $\sim 1/137$ observed in four dimensions. The restrictions (16) and (14) are inherent in all higher dimensional theories and are due to the fact that the action of the gauge fields is derived from the Einstein action in the higher dimensional space-time, in which only the gravitational constant plays the role of a dimensional constant. If one has some reservation regarding setting the scale R_s through (16) and (14), it is better to consider a theory in which R_s is a free parameter. We adopt the induced gravity formalism (Naka and Itoi, 1983) in which the theory of gravity is defined as a quantum fluctuation of fundamental matter fields. We start with the action (also see Sivaram, 1987)

$$S=\int d^5x\,|f|^{1/2}\cdot\tfrac{1}{2}(f^{\hat{\mu}\hat{\nu}}\,\partial_{\hat{\mu}}\phi^i\,\partial_{\hat{\nu}}\phi^i-m^2\phi^i\phi^i) \tag{17}$$

where $\hat{\mu}$, $\hat{\nu}=0, 1, 2, 3, 5$; $f^{\hat{\mu}\hat{\nu}}f_{\hat{\nu}\hat{\rho}}=\delta^{\hat{\tau}}_{\hat{\rho}}$; and $f=\det(f_{\hat{\mu}\hat{\nu}})$. We assume that:

(i) The scalar fields ϕ^i satisfy the periodic boundary condition with respect to internal coordinate x^5 such that

$$\phi^i(x^5+2\pi R_s)=\phi^i(x^5) \tag{18}$$

where R_s is a constant having the dimension of length.

(ii) The auxiliary fields $f_{\hat{\mu}\hat{\nu}}$ can be decomposed as

$$f_{\hat{\mu}\hat{\nu}}=\left(\begin{array}{c|c} f_{\mu\nu}-(eR_s)^2A_\mu A_\nu & (eR_s)A_\nu \\ \hline (eR_s)A_\nu & -1 \end{array}\right) \tag{19}$$

where $f_{\mu\nu}$ and A_μ are functions of x^μ only and will be understood as the metric tensor of strong gravity and electromagnetic field, respectively. The action (17) now becomes

$$S=\int d^5x\,|f|^{1/2}\,\tfrac{1}{2}[f^{\mu\nu}(\partial\mu+eR_sA_\mu\,\partial_5)\phi^i(\partial_\nu+eR_sA_\nu\,\partial_5)\phi^i$$
$$+\partial_5\phi^i\,\partial_5\phi^i-m^2\phi^i\phi^i] \tag{20}$$

The boundary condition (18) again allows us to write $\phi^i(x^\mu, x^5=\phi)$ in the form

$$\phi^i(x^\mu, x^5)=\frac{1}{(2\pi R_s)^{1/2}}\sum_{n=-\infty}^{+\infty}\phi^i_n(x^\mu)\,e^{inx^5/R_s}$$

where $\phi^i_{-n}=\phi^{i*}_n$. Integrating over x^5, we get (20) as

$$S=\sum_{n=-\infty}^{+\infty}\int d^4x\,|f|^{1/2}\cdot\tfrac{1}{2}[f^{\mu\nu}(\partial_\mu\phi^i_{-n}+ie_nA_\mu\phi^i_{-n})$$
$$\times(\partial_\nu\phi^i_\nu+ie_nA_\nu\phi^i_n)-m_n^2\phi^i_{-n}\phi^i_n] \tag{21}$$

where $e_n=en/Rs$, $m_n^2=m^2+n^2/R_s^2$, and $f=\det(f_{\mu\nu})$. The effective action for $f_{\mu\nu}$ and A_μ is now obtained as

$$W_{\text{eff}}=i\frac{N}{2}\,\text{Tr}\sum_n\log\left(\frac{i\hat{H}_n}{2\pi\mu^2}\right)$$

where

$$\hat{H}_n(x)=\frac{1}{|f|^{1/2}}(\partial_\mu+ie_nA_\mu)|f|^{1/2}f^{\mu\nu}(\partial_\nu+ie_nA_\nu)+m_n^2$$

μ is a normalization constant having dimension of mass, and N is the number of scalar fields.

The effective action now turns out to be

$$W_{\text{eff}} = \int d^4x \, |f|^{1/2} \left(\bar{c}_0 - \bar{c}_1 \frac{R}{6} - \frac{e^2 R_s^2}{16} \bar{c}_2 f^{\mu\nu} f^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} + \cdots \right) \tag{22}$$

$$\bar{c}_0 = i \frac{N}{16\pi^2} \operatorname{tr} \left\{ P_5^4 \left[-\log \left(\frac{P_5^2}{-2\pi i \mu^2} \right) + \frac{3}{2} \right] \right\}$$

$$\bar{c}_1 = \frac{N}{32\pi^2} \operatorname{tr} \left\{ P_5^2 \left[-\log \left(\frac{P_5^2}{2\pi i \mu^2} \right) + 1 \right] \right\}$$

$$\bar{c}_2 = \frac{N}{32\pi^2} \operatorname{tr} \left[-P_5^2 \log \left(\frac{P_5^2}{-2\pi i \mu^2} \right) \right]$$

Here the trace runs over the eigenvalues of P_5 ($=0, \pm 1/R_s, \pm 2/R_s, \ldots$). We assume also that ultraviolet divergence contained in the trace part is suitably regularized by an internal momentum cutoff, thus making $\bar{c}_0$, $\bar{c}_1$, and $\bar{c}_2$ finite constants.

We adjust the effective action by putting

$$\tfrac{1}{2}\bar{c}_1 = \frac{e^2 R_s^2}{16\pi G_s} \bar{c}_2$$

so that

$$W_{\text{eff}} = \int d^4x \, |f|^{1/2} \left(\bar{c}_0' + \frac{1}{16\pi G_s} R + \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots \right) \tag{23}$$

where

$$\bar{c}_0' = -(3/8\pi G_s)(\bar{c}_0/\bar{c}_1)$$

This completes our derivation. Thus we have generated a large cosmological constant arising out of massive modes $\sim n/R_s$, fixing R_s to be of the order of 10^{-13} cm, the characteristic length of the strong interaction. Thus, the use of a large cosmological constant in strong gravity finds a justification in our approach. This is a new result (see also Sivaram and Sinha, 1973, 1976).

4. MASS SCALES AND REGGE STATES

From the above discussion it is clear that the massive states of the theory are given by

$$M_n = n/R_s \tag{24}$$

where n is an integer. However, it is quite possible that there is a fluctuation of internal space with time. In that case we have to take the average over time and the relation (24) will be modified as

$$M(k)=K/R_s \tag{25}$$

where K is not necessarily an integer. It was noted by Møller (1952, p. 170) that a classical system with positive energy density, a given intrinsic angular momentum, and a given rest mass must always have a finite extension given by

$$R_s=J/M \tag{26}$$

We have already noted that $R_s \simeq$ the dimension of a hadron, so that the striking resemblance of (25) and (26) and dimensional arguments allows us to identify K with the intrinsic angular momentum J and we get

$$J=R_sM \tag{27}$$

Thus we see that the internal space generates masses in such a way that (i) it simulates a peripheral behavior $J=\alpha(s)\propto\sqrt{s}$, and (ii) it produces approximately the spectrum as expected from dual models. It should be noted that Regge theory with peripheral dominance gives rise to trajectory $\alpha(s)\subset\sqrt{s}$ in the s-channel description (Schrempp and Schrempp, 1977). So the f-gravity theory in five dimensions is able to explain the origin of the hadronic mass spectrum without solving an Einstein-like equation of strong gravity and using the soliton mass formula of Christodoulo and Ruffini (1971). One might be skeptical of the arguments leading to (27) and inquire about the role of the field equations in the theory. The f–g theory in five dimension under dimensional reduction will lead to field equations

$$R_{\mu\nu}(f)+\tfrac{1}{2}f_{\mu\nu}R+\Lambda f_{\mu\nu}=0 \tag{28}$$

The Ricci tensor $R_{\mu\nu}(f)$ is evaluated with the metric $f_{\mu\nu}$. Incorporating internal symmetry and Yang–Mills type source terms for color field, one is able to generate a mass formula (Mielke, 1980) for hadrons (also see Sivaram and Sinha, 1977)

$$\frac{M^2}{M^{*2}}=\left\{1+\beta Y+\frac{\alpha}{4}[I(I+1)+\tfrac{1}{4}Y^2]\right\}^2+\tfrac{1}{4}J(J+1) \tag{29}$$

where the symbols have their usual meaning. M^* is the Planck mass, ~1 GeV, of strong gravity and α is the coupling of the Yang–Mills field to strong gravity; β is fitted numerically ($\simeq\frac{1}{3}$). A simple approach is to consider hadrons of mass M as black holes of radius $R_s=G_sM$, so that equation (25) now generates a linearly rising trajectory $J=\alpha(s)\to\alpha' M^2$ with $\alpha'=G_s$. The result is quite convincing. It is well known that the existence of a linearly rising trajectory is a direct manifestation of the confining aspects of hadronic constituents. As we increase energy, all resonance states lie on a Regge trajectory without having any ionization energy to set the constituents free (Sivaram and Sinha, 1979).

5. TRAPPING OF QUARKS AND GLUONS

In this section we discuss the trapping of quarks and gluons in the given gravitational background of 5D f–g theory. The 5D f–g theory reduced to four dimensions is equivalent to the Einstein action with a cosmological constant. The field equation turns out to be

$$R_{\mu\nu}(f)+\tfrac{1}{2}f_{\mu\nu}R+\Lambda_f f_{\mu\nu}=-T_{\mu\nu}(\phi) \tag{30}$$

where

$$T_{\mu\nu}(\phi)=\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}f_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}$$

is the energy-momentum tensor of scalar quarks with

$$f^{\mu\nu}\phi_{;\mu\nu}=0 \tag{31}$$

We assume

$$f_{\mu\nu}=e^{2\lambda(r)}\eta_{\mu\nu} \tag{32}$$

and $e^{2\lambda}\to 1$ as $\gamma\to 0$, so that quarks are free at the center of confinement $(r=0)$. After solving (30) with the form (32), we get

$$e^{2\lambda(r)}=1-\Lambda_f r^2/3 \tag{33}$$

Now assuming gluons as quasi-Maxwellian fields, the gluon field equations in the background (33) are written as (Landau and Lifshitz, 1975), p. 334)

$$\begin{aligned}\nabla\cdot\mathbf{D}&=0\\ \nabla\cdot\mathbf{B}&=0\\ \nabla\times\mathbf{H}&=\partial\mathbf{D}/\partial t\\ \nabla\times\mathbf{E}&=-\partial\mathbf{B}/\partial t\end{aligned} \tag{34}$$

where $\mathbf{D}=e^{2\lambda}\mathbf{E}$ and $\mathbf{B}=e^{2\lambda}\mathbf{H}$. Thus we see that the $f_{\mu\nu}$ field plays the role of a medium with

$$\varepsilon=\mu=1-\Lambda_f r^2/3 \tag{35}$$

The characteristic features of the solution of (34) are that (i) the energy of the gluons are quantized, $E(n)\propto(n+\tfrac{3}{2})^{1/2}$ with $n=0, 1, 2, \ldots$, and (ii) $\mathbf{E}$ and $\mathbf{B}$ are confined in a region $\Lambda_f^{1/2}r\ll 1$ and higher energy states are closer to the center. This signals confinement in the theory.

To show confinement of quarks, we take the defining equations of massless quarks in the gravitational field as

$$(\boldsymbol{\sigma} \cdot \mathbf{p})\phi = \omega \in \chi$$

$$(\boldsymbol{\sigma} \cdot \mathbf{p})\chi = \omega \in \phi$$

We solve (35) for $\Lambda_f^{1/2} r \ll 1$ and find, as in the color field case, that quark energies are spaced like harmonic oscillator-like levels and the exponential decrease of the radial part of the solution of (35) reveals that higher energy quark orbits are closer to the center of confinement. The conclusion is that the strong gravitational background provides a trap for both gluons and quarks. The details will be discussed in a subsequent paper.

6. EFFECT OF EXPANDING INTERNAL SPACE

In order to establish our claim, namely equation (25), we consider an internal space with expanding internal space with Robertson-Walker-type metric,

$$\begin{aligned} ds^2 &= dt^2 - R_s^2(t)\, d\phi^2 \\ &= c(\eta)(d\eta^2 - d\phi^2) \end{aligned}$$

where

$$t = \int dt = \int R_s(\eta')\, d\eta'$$

Let

$$\begin{aligned} c(\eta) &= R_s^2(t) \\ c(\eta) &= A + B \tanh \rho\eta \end{aligned} \tag{36}$$

with A, B, ρ constants. So

$$c(\eta) = A \pm B \qquad \text{for} \quad \eta \to \pm\infty \tag{37}$$

The field equation corresponding to the Lagrangian density

$$\mathscr{L}(x) = \tfrac{1}{2}[-f(x)]^{1/2}[f^{\mu\nu}(x)\phi_{,\mu}(x)\phi_{,\nu}(x) - m^2\phi^2(x)]$$

is

$$\frac{d^2}{d\eta^2} x_k(\eta) + \left[\frac{n^2}{R_s^2} + c(\eta)m^2\right]\chi_k(\eta) = 0$$

where

$$\phi(x)=\sum_i a_i U_i(x)+a_i^\dagger U_i^*(x)$$

$$U_k(\phi, n)=(2\pi)^{-1/2}\, e^{i(n/R_s)\phi}\chi_k(\eta)$$

It is found that $U_n^{\text{in}} \neq U_n^{\text{out}}$, where

$$U_n^{\text{in}} \xrightarrow[\eta\leftarrow-\infty]{} (4\pi\omega_{\text{in}})^{-1/2}\, e^{i(n/R_s)\phi - i\omega_{\text{in}}\eta}$$

$$U_n^{\text{out}} \xrightarrow[\eta\rightarrow+\infty]{} (4\pi\omega_{\text{out}})^{-1/2}\, e^{i(n/R_s)\phi - i\omega_{\text{out}}\eta}$$

$$\omega_{\text{in}}=\left[\frac{n^2}{R_s^2}+m^2(A-B)\right]^{1/2}$$

$$\omega_{\text{out}}=\left[\frac{n^2}{R_s^2}+m^2(A+B)\right]^{1/2}$$

Defining Bogolubov coefficients α_n and β_n as (Birrell and Davies, 1982, p. 59)

$$U_n^{\text{in}}(\eta, x)=\alpha_n U_n^{\text{out}}+\beta_n U_{-n}^{\text{out},*}$$

we find that $|\beta_n|^2 \neq 0$, implying thereby that there is mass creation in such an expanding internal space. If the rate of expansion is slow,

$$|\beta_n|^2 \propto B^2$$
$$\propto R_s\,\Delta R_s$$

For the hadronic regime, due to the large value of G_f, the mass generation will be significant. However, when the slowness parameter $\rho\rightarrow 0$,

$$|\beta_n|^2 \propto e^{-M_n\rho}$$

so that higher modes are inefficiently excited; here $M_h \simeq$ mass of a hadron. Thus, the internal momentum cutoff used in Section 3 is quite reasonable in our approach. Further, the energy of massive states is now given by

$$\omega^2 \simeq n^2/R_s^2+m^2(A\pm B)$$

With $m^2=\text{const}/R_s^2$, we get

$$\omega^2=J^2/R_s^2$$

where $J^2=n^2+\text{const}\cdot(A\pm B)$. Equation (38) justifies our claim that the massive states of the theory lie on a Regge trajectory. The details of this section will be discussed elsewhere.

7. CONCLUSION

In normal gravitation the cosmological constant is approximately zero, whereas we require a large cosmological constant Λ_f in the strong gravity formalism. In this paper, we have tried to deal with the occurrence of the large cosmological constant starting from a five-dimensional Kaluza-Klein theory. In all strong gravity theories the solitonic solutions of field equations are identified as hadrons, thereby hiding the dynamical content of the confinement mechanism. In this paper we have also tried to deal with the way the confinement is achieved in the theory, and Regge states, a direct manifestation of confinement, find a natural place in our approach.

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